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Hugoniot Equations of State for Plastics: A Comparison

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The Hugoniot equation of state is one of the constitutive relations describing the response of a material to hypervelocity impact. In recent years, impact at hypervelocities (in excess of 15,000 fps) has assumed a more important role because of the exposure of artificial satellites to impact of micrometeorites and because of the possible use as a concept of defensive and offensive missile systems. (Most hypervelocity impact experiments using the flying plate are extremely costly to conduct; therefore, a number of attempts have been made to correlate empirically and then to extrapolate the experimental data.) This paper compares these empirical equations of state for some plastics.

The term equations of state (sometimes called constitutive relations) means the relationships between the physical variables that are characteristic of the material involved. In gasdynamics, there is only one constitutive relation, the pressure-volume-temperature relation. In solid mechanics, the most general constitutive relations express the functional relationships among the stress components, strain components, stress-rate components, strain-rate components, and the specific internal energy. To arrive at a workable equation of state, several simplifying assumptions must be made.

If the stress state for which the equation of state is determined is assumed to be hydrostatic (i.e., uniform compressive stress), the appropriate equation of state is one in which the hydrostatic pressure replaces the stress components and the density (or compression ratio) replaces the strain components. Since the stress and strain components associated with any general state of stress can be broken into two components, dilatational (simple expansion or contraction) and distortional, the hydrostatic equation of state can be thought of as a part of the general stress-strain-temperature relations.

This assumption of hydrostatic condition is a reasonable one, since, in the initial phase of hypervelocity impact of two solids, the stresses are very much greater than the yield stress in shear so that at least for this time phase, a hydrostatic Hugoniot equation of state appears to be pertinent. Therefore, a hydrostatic Hugoniot equation of state will be referred to simply as the "equation of state" in this paper.

Most empirical equations of state are approached in the following manner. The conditions of conservation of mass and of momentum across a shock front (sometimes called the Rankine-Hugoniot relations) can be written as follows:

$$\rho/\rho_0 = U_s/(U_s - u_p) \qquad \text{(conservation of mass)}$$

$$P - P_0 = \rho_0 U_s u_p \qquad \text{(conservation of momentum)}$$

where ρ is the density, U_s is the shock velocity, u_p is the particle velocity, P is the pressure, and the subscript 0 denotes initial conditions. These relations assume that the initial velocity U_0 equals zero.

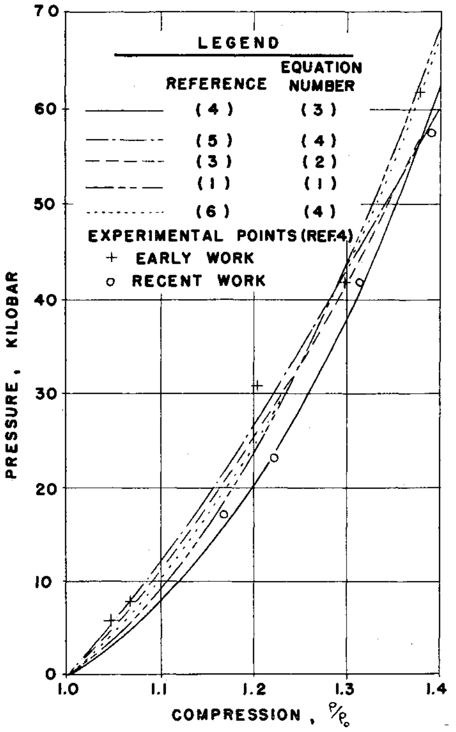

Fig. 1. Pressure-compression curves for polymethylmethacrylate including experimental data.

The relationship between the shock and particle velocities is generally assumed to be a linear one and has been found experimentally to be applicable to a variety of materials. This relationship is of the form

$$U_s = U_0 + \lambda u_p$$

where U_0 and λ are constants for a given material. The preceding expressions are then combined to develop an equation of state where the pressure $P = f(R)$ and $R = \rho/\rho_0$. In the following discussion, the constants are chosen to express P in kilobars.

Equations of State

In recent years, a number of seemingly different, empirical equations of state have been advanced. These equations are presented in chronological order with respect to date of publication. In 1958, Buchanan, James, and Teague[1] examined Perspex (a methyl methacrylate polymer) in compression and arrived at the following empirical expression:

$$P = 66.4(R - 1) + 262(R - 1)^2 \tag{1}$$

The authors claim that, in 1956, Lawton and Skidmore[2] presented experimental evidence in support of their expression. In 1961, Blewett[3] examined both Lucite (a methyl methacrylate polymer) and polyethylene and found that his results fitted this somewhat simpler expression:

$$P = A(R^\gamma - 1) + 10^{-3} \tag{2}$$

where for Lucite, $A = 30.1$, $\gamma = 3.33$, and for polyethylene, $A = 2.30$ and $\gamma = 9.20$.

In 1962, three new empirical expressions for the equations of state were published. The first was presented by Wagner, Waldorf, and Louie[4] in the form

$$P = BR(R - 1)/(C - R)^2 \tag{3}$$

The authors investigated numerous plastics and determined

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Table 1 Material constants for equation of state[4]

Material	B, kbar	C, dimensionless
Polyethylene	11.9	1.73
Kel-F	170	2.65
Plexiglas	217	2.8
Nylon	154	2.6
Polystyrene	230	2.66
Teflon	45.1	2.08

corresponding values for the constants B and C as shown in Table 1.

The second publication in 1962 was by Barry, Semon, Berry, Lucy, and Flom[5] who investigated several polymers themselves and used some data from Ref. 4 on Lucite to develop the equation

$$P = DR_u/(1 - \lambda R_u)^2 \qquad (4)$$

The authors used a velocity ratio R_u instead of a density ratio R where

$$R_u = \frac{u_p}{U_s} = \frac{\text{particle velocity}}{\text{shock wave velocity}}$$

or

$$R_u = (R - 1)/R \qquad (5)$$

The materials and corresponding constants presented by the authors are listed in Table 2.

Duvall and Fowles[6] also used the form of Eq. (4) in a tabulation of constants for a large number of materials including a methacrylate polymer where the values $D = 88.6$ kbar and $\lambda = 1.35$ are given. The third equation presented in 1962 was that of Guenther,[7] which is in the form

$$P = F(R - 1) + G\,(R - 1)^2 + HRE \qquad (6)$$

where F, G, H are constants, and E is the internal energy per unit volume. The author presents values for the constants for six different plastics, but gives no value for the internal energy.

A Comparison

From the previous section, it is seen that most of the authors have presented data on a methyl methacrylate polymer (Lucite, Perspex, Plexiglas). Therefore, this material can be used as a basis for comparing the various empirical equations of state. Figure 1 shows this comparison using Eqs. (1–4). In addition, individual experimental points were available from Ref. 4 and these are included in the figure.

Figure 2 shows a comparison of some data obtained at pressures over 100 kbar by a radiographic technique.[4, 8] As seen in Fig. 2, there does not appear to be continuity between the data obtained at the higher pressures and those obtained at lower pressures. With the large amount of data available at the lower pressures and the scarcity of data at the higher pressures, it is apparent that further experimental studies are necessary. The experiments should incorporate different experimental techniques in the pressure range above 100 kbar in order to clarify the apparent discrepancy with data for lower pressures.

Table 2 Material constants for equation of state[5]

Material	D, kbar	λ dimensionless
BC 323[a]	54.5	4.48
123-C,[a] 124[a]	73.2	1.8
Lucite	108	1.05

[a] Castable resins phenolic substructure with glycidyl ether groups attached

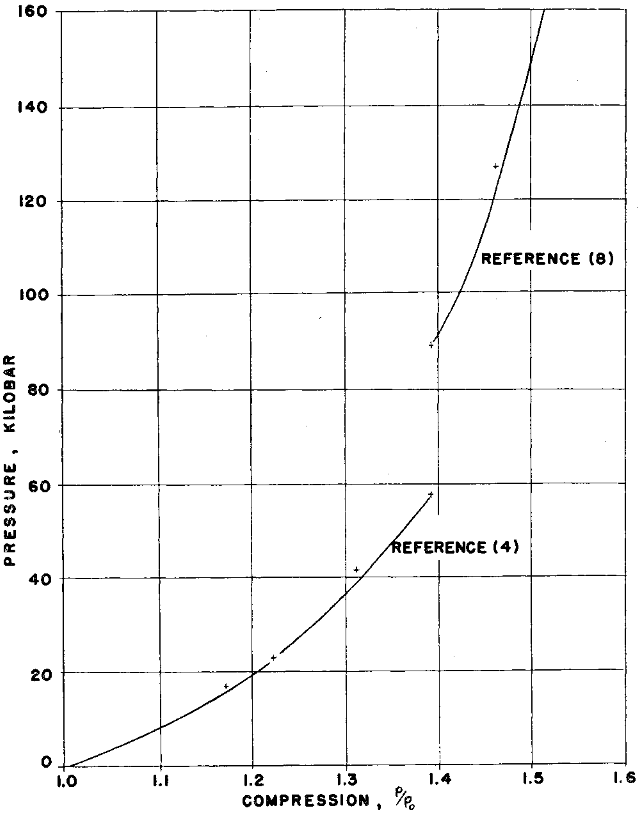

Fig. 2. **Pressure-compression curves for polymethylmethacrylate.**

Conclusions

As can be seen in Fig. 1, there is very little difference between the forms of representation for the equation of state and the experimental data over the pressure range covered.

Therefore, it would be advantageous to use the form that is the most adaptable to the data available or that happens to be preferred by the investigator.

In addition, and more importantly, there is a need for the development of more general constitutive relations, and to investigate materials at higher pressures than those published to date.

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